

Anomalous Appearance Of The World At High Velocities: A Mathematical Approach

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Abstract –According to “ Special Theory of Relativity ”, Time and space appear different to observers in relative motion. When an object is in motion its length gets contracted, this contraction is known as "Lorentz Contraction". “ Lorentz Contraction ” occurs in the direction of motion of the observer and no contraction occurs in the direction other than that of motion of the observer. But, does this causes any effect on the appearance of the world around the moving observer? Thus, my aim is: to prove that Lorentz Contraction has a strange effect on the appearance of the world and devise mathematical understanding of it. Through our methodology, we have achieved the result that when an observer is moving with constant velocity with respect to the space around it, the space ahead of him appears to be curving or warping in and the space behind him appears to be curving or warping away. We also derived the equations of Apparent path, whose one of the corollary is the perspective of photon, which says that the world appears to be stopped to a photon. In short, We have concluded that there exists a contraction and an unexpected expansion of space as viewed by a person in constant motion.

Our result and equations can be used in many areas of high motion imaging. For instance, it can be used to increase the accuracy of "Radar targeting of fighter jets".

Index Terms– Appearance of world to a photon, Appearance Of World at high velocities, Contraction and expansion of space, Disappearing of world at speed of light, increase of missile targeting accuracy, Special Relativity, Problem of sixth gen. jet's missile targeting.

1 INTRODUCTION

WHEN I was considering why, when we are moving with constant velocity, the objects appear to be coming towards us with a velocity which keeps on increasing or an accelerated velocity, i figured out that it is because of line of sight. Then, i calculated the rate of change of angle of sight with respect to time. I found out that the rate of change is not constant. Then, a thought came in my mind that how can the angle attached to the body moving with constant velocity, be accelerated. This means the world should adapt itself to make the angle's rate of change constant (which is quite wrong). Then, i got on the endeavour of finding how the world changes. I first claimed that the world curves inwards, because rate of change of central angle in a circle is constant. Though, quite interesting, i got my way through Special relativity.

Special relativity was introduced in Einstein's 1905 paper "On the Electrodynamics of Moving Bodies". Special relativity is based on two postulates: 1. The laws of physics are the same for all observers in uniform motion relative to one another (principle of relativity). 2. The speed of light in a vacuum is the same for all observers, regardless of their relative motion or of the motion of the source of the light.

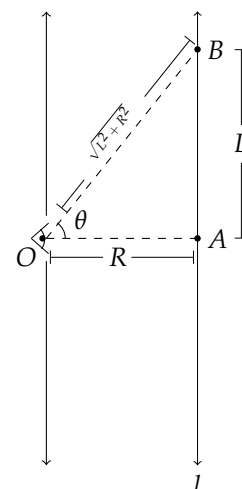
Special relativity says that, when an object is in motion it's length get contracted and time on clocks attached to it slows down. The length contraction is known as "Lorentz Contraction" and the slowing of time is known as "Time Dilation". It also says that the clocks attached to that moving object are "Asynchronous" in nature, the ones at front

of the object, along the direction of motion of the object, are behind those which are at back of the object. I used these results of relativity to prove that there exists a special type of contraction and expansion of space.

2 METHODOLOGY

Let us suppose that we are in a train standing on a platform.

Consider a wall at R units away parallel to train, named by line l . Let the observer be at point O and there be a perpendicular going out of train from point O , intersecting the wall at a point A . Let there be a point B , on that wall, L units above A .



Let the train be stationary, for now.

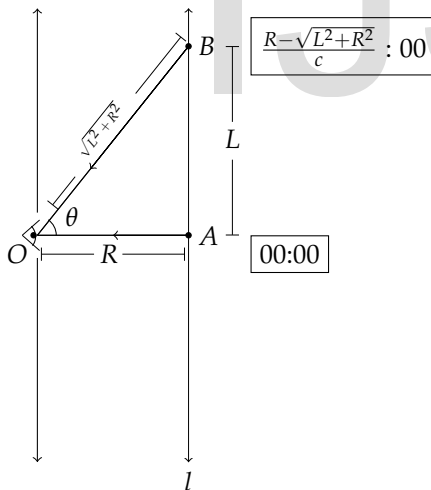
$$\text{Time taken by light ray from A to reach O} = \frac{R}{c}$$

$$\text{Time taken by light ray from B to reach O} = \frac{\sqrt{L^2 + R^2}}{c}$$

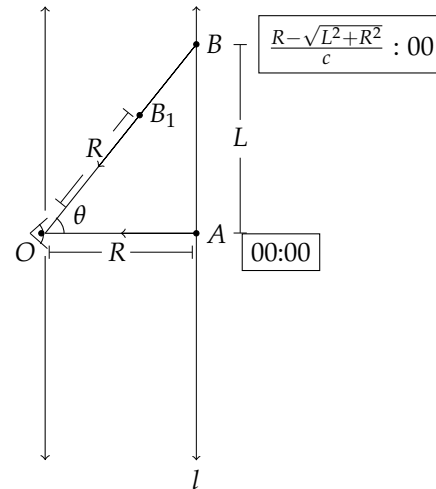
This means that if we eject both light rays simultaneously, we will be receiving the light rays from A and B after $\frac{R}{c}$ and $\frac{\sqrt{L^2 + R^2}}{c}$ units, respectively, of their emission, at O. So, the time difference in their receiving, at O, will be $\frac{\sqrt{L^2 + R^2} - R}{c}$ units.

So, if we eject the light ray from B at time $\frac{\sqrt{L^2 + R^2} - R}{c}$ units before that ejected from A, i.e. if there is a time difference of $\frac{\sqrt{L^2 + R^2} - R}{c}$ in between the emissions of light rays from A and B respectively. Then, we will be receiving both of them simultaneously at O.(1)

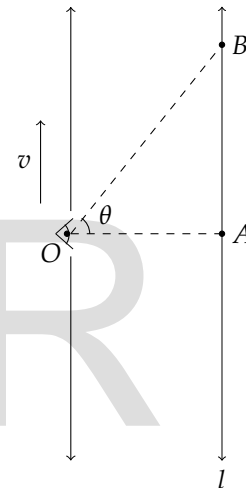
For instance, if the light ray from B is ejected at $\frac{R - \sqrt{L^2 + R^2}}{c} : 00$ and that from A is ejected at 00:00. Then, they will be received simultaneously at O.



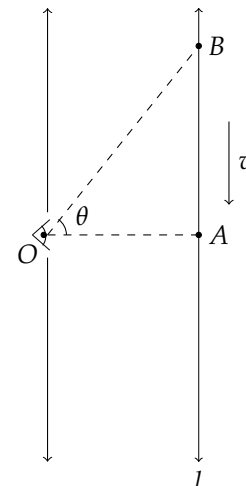
We will be receiving both of them simultaneously at O because light ray from B is ejected earlier than that from A. Therefore, the light ray from B has already travelled some distance and had reached a point B₁ before the light ray from A had been ejected and the remaining distance B₁O is equal to R, such that the time light takes to travel B₁O is equal to that it takes to travel AO. Hence, both are received simultaneously at O.



Now, let the train be moving with velocity v in upward direction.



As the train is moving with constant velocity v in upward direction, we can say that the objects around train including the wall are moving with velocity v in downward direction with respect to train.



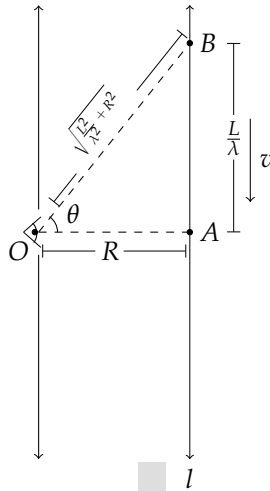
According to Special Theory Of Relativity (STOR), when a body is in motion its length contracts in the direction of

motion of body, which is known as "Lorentz Contraction".
The contraction factor is λ , such that

$$\lambda = \sqrt{1 - \frac{v^2}{c^2}}$$

where v = speed of body and c = speed of light

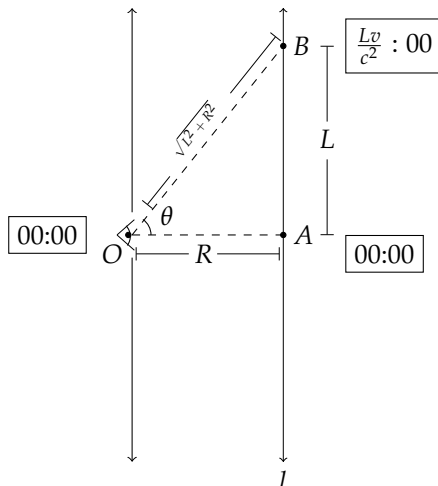
Since, line l is in motion, with respect to (w.r.t.) the observer in train, line l' 's length will contract by a factor of λ . So, it will become $\frac{L}{\lambda}$.



STOR also says that clocks attached to the body in motion are asynchronous in nature by a factor of $\frac{Lv}{c^2}$, where L is the distance between clock. Also the time in clocks at the points of intersection of the perpendicular line joining point of observation to the line of motion is same.

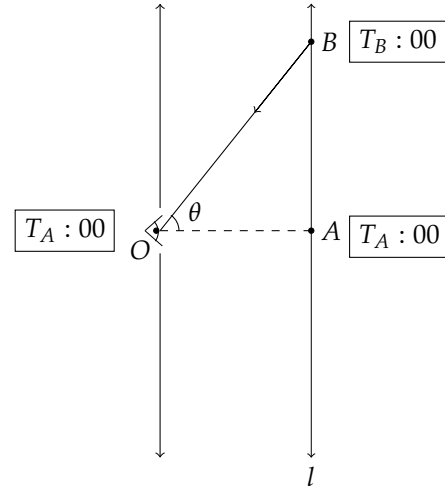
So, the clocks on line l will be asynchronous. The ones towards B being ahead than those towards A , by a factor of $\frac{Lv}{c^2}$. Also, the time at A and O is same.

Let the time at A be 00 : 00. Then, time at B and O will be $\frac{+Lv}{c^2} : 00$ and 00 : 00 respectively.

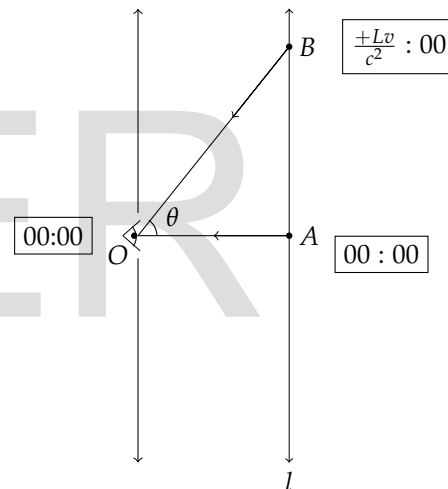


Let when the time at B is $T_B = \frac{R}{c} - \frac{1}{c} \sqrt{\frac{L^2}{\lambda^2} + R^2}$ a light ray is ejected from there towards O . Since, B is ahead of

A by $\frac{Lv}{c^2}$ units. Therefore, time at A will be $T_A = \frac{R}{c} - \frac{1}{c} \sqrt{\frac{L^2}{\lambda^2} + R^2} - \frac{Lv}{c^2}$ and so will be at O .



Now, let a light ray be ejected from point A when time at A is 00 : 00. So, the time at B and O will be $\frac{+Lv}{c^2}$ and 00 : 00 respectively.

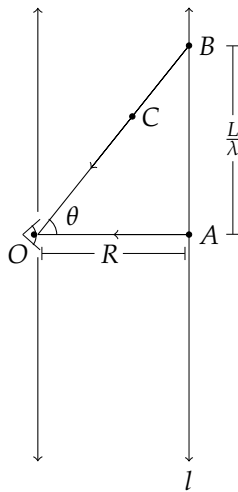


As the light ray from B has ejected earlier than that from A . So, The light ray from B had travelled some distance before the light ray from A has been ejected. Let the light ray from B be at C at the time when light ray from A has been ejected.

Consider the previous case when the train was stationary, we found out that if there is a time difference of $\frac{\sqrt{L^2 + R^2} - R}{c}$ in between the ejections of light rays from A and B respectively. Then, we will be receiving both of them simultaneously at $O(1)$. But, how do we have measured the time difference? We measured the time of projection of light ray from B through a clock situated at B and measured the time of projection of light ray from A through a clock situated at A . Then, we took the difference of the time.

Now, we will similarly consider the time difference in the second case. The time of projection of light ray from B is $\frac{R}{c} - \frac{1}{c} \sqrt{\frac{L^2}{\lambda^2} + R^2}$ (as measured by a clock at B)

and that of A is 00 : 00 (as measured by a clock at A). So, the time difference is $\frac{\sqrt{\frac{L^2}{\lambda^2} + R^2} - R}{c}$. But, it is the time we provided in (1). So, we should be seeing both the light rays simultaneously at O. So, $CO=R$.



Now, time difference between ejection of light rays from B and A, as measured by a clock at B, is $\frac{Lv}{c^2} + \frac{1}{c}\sqrt{\frac{L^2}{\lambda^2} + R^2} - \frac{R}{c}$. Light ray from B should travel the distance CB in this time period.

So,

$$BC = c \left(\frac{Lv}{c^2} + \frac{1}{c}\sqrt{\frac{L^2}{\lambda^2} + R^2} - \frac{R}{c} \right)$$

$$= \frac{Lv}{c} + \sqrt{\frac{L^2}{\lambda^2} + R^2} - R$$

And

$$CO = BO - BC \quad (3)$$

$$CO = \sqrt{\frac{L^2}{\lambda^2} + R^2} - \left(\frac{Lv}{c} + \sqrt{\frac{L^2}{\lambda^2} + R^2} - R \right)$$

$$CO = R - \frac{Lv}{c}$$

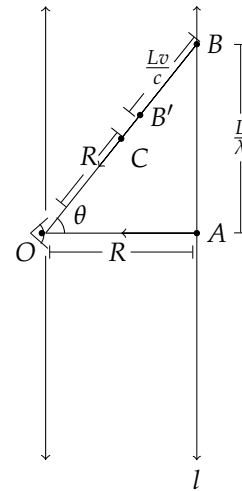
But, $CO = R$.

Now, Two things can happen either (1) is wrong, which can only happen when light's speed is not constant, so light should vary its speed such that the time taken by light from point A and point B to reach O becomes same, i.e. time taken by light to travel R and $R - \frac{Lv}{c}$ is same or the distance BO appears to be reduced by length $\frac{Lv}{c}$. Such that B will appear to be situated at a point B', so the length B'B, OB' and CO is equal to $\frac{Lv}{c}$, $\sqrt{\frac{L^2}{\lambda^2} + R^2}$ and $CO = R$, respectively. So,

$$BO = OB' + B'B = \sqrt{\frac{L^2}{\lambda^2} + R^2} + \frac{Lv}{c}$$

Putting, this value in eq.(3), we get $CO = R$. Hence, (1) still remaining true.

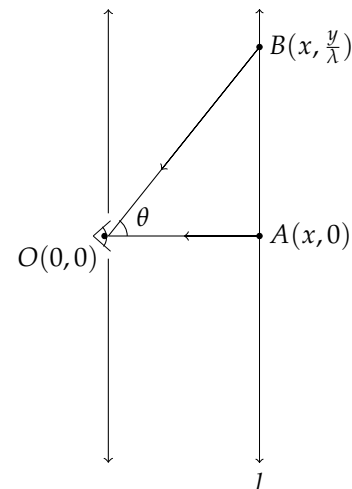
But, as Maxwell proved that speed of light is constant. So, BO should appear to be reduced by length $\frac{Lv}{c}$.



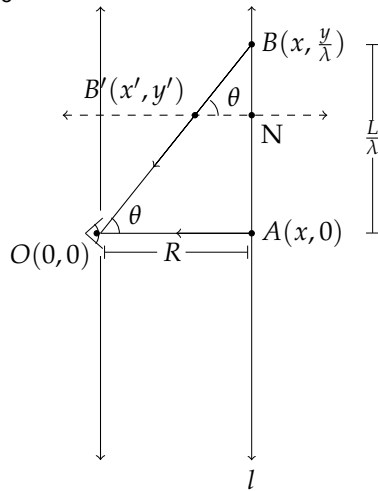
This doesn't only applies to B, but on every single point lying on the every single line parallel to the path of motion of observer. Each of them appears to be bent inwards or get close by a factor of $\frac{Lv}{c}$.

3 MATHEMATICAL MODELLING

Let the observer be at (0,0) and point A(x,0) and B(x,y) lie on the line $X = x$. Let the angle of sight be θ .



Let the line start to go backward with a velocity v. Then, by STOR y becomes $\frac{y}{\lambda}$ and x remains the same. So, coordinates of B are $(x, \frac{y}{\lambda})$. Let the coordinates of the point B' be (x', y') . Now, draw a line $Y = y'$ let it intersect $X = x$ at N.



As we have found earlier that $BB' = \frac{Lv}{c}$ and angle of sight is θ . Therefore,

$$B'N = \frac{Lv}{c} \cos \theta \quad \text{And} \quad BN = \frac{Lv}{c} \sin \theta$$

So,

$$x' = x - \frac{Lv}{c} \cos \theta \quad \text{And} \quad y' = \frac{y}{\lambda} - \frac{Lv}{c} \sin \theta \quad (4)$$

But,

$$\Rightarrow \sin \theta = \frac{BA}{BO} = \frac{\frac{y}{\lambda}}{\sqrt{\frac{y^2}{\lambda^2} + x^2}}$$

$$\sin \theta = \frac{y}{\sqrt{y^2 + \lambda^2 x^2}}$$

$$\Rightarrow \cos \theta = \frac{AO}{BO} = \frac{x}{\sqrt{\frac{y^2}{\lambda^2} + x^2}}$$

$$\cos \theta = \frac{x\lambda}{\sqrt{y^2 + \lambda^2 x^2}}$$

Putting this value of $\sin \theta$ and $\cos \theta$ in eq. (4), we get

$$\Rightarrow y' = \frac{y}{\lambda} - \frac{y^2 v}{c \sqrt{y^2 + \lambda^2 x^2}}$$

$$x' = x - \frac{y v x \lambda}{c \sqrt{y^2 + \lambda^2 x^2}}$$

$$\Rightarrow y' = y \left(\frac{1}{\lambda} - \frac{y v}{c \sqrt{y^2 + \lambda^2 x^2}} \right)$$

$$x' = x \left(1 - \frac{y v}{c \sqrt{y^2 + \lambda^2 x^2}} \lambda \right) \quad (5)$$

Let $M = \frac{yv}{c \sqrt{y^2 + \lambda^2 x^2}}$
So, the equation of apparent path is,

$$\Rightarrow y' = y \left(\frac{1}{\lambda} - M \right)$$

$$x' = x (1 - M\lambda)$$

$$\text{, Where } M = \frac{yv}{c \sqrt{y^2 + \lambda^2 x^2}} \quad (6)$$

Further, simplifying our equations we get,

$$y' = \frac{y}{\lambda} (1 - M\lambda) \quad (7)$$

And,

$$x' = x(1 - M\lambda) \Rightarrow \frac{x'}{x} = (1 - M\lambda) \quad (8)$$

Putting value of $(1 - M\lambda)$ in eq. (7) from (8), we get,

$$\Rightarrow y' = \frac{y}{\lambda} * \frac{x'}{x}$$

$$\Rightarrow \frac{y'}{y} = \frac{1}{\lambda} * \frac{x'}{x}$$

$$\Rightarrow \frac{y'}{x'} = \frac{y}{x}$$

which says that slope of both points is equal. Hence, both will lie on the same line of sight.(9)

4 OBSERVATION

Let us consider two walls on both sides of the train, then they will appear differently as provided by our equation

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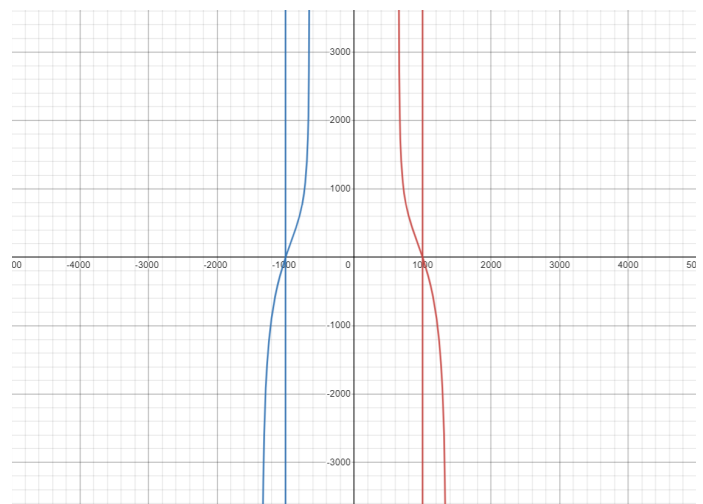


Figure 1: The graph of apparent path equation by DESMOS, here the straight lines show the appearance of walls at rest and the curved lines show their appearance when the observer is in motion

It is clearly evident from the graph of our apparent path equation that when we are moving with a constant velocity, the world ahead of us appears to be curving inwards and the world behind us appears to be curving outwards.

Special cases:

(i) For $v = 0$, our equations give $y' = y$ and $x' = x$. Which means that we will be seeing the walls as they are, as given in Fig.2

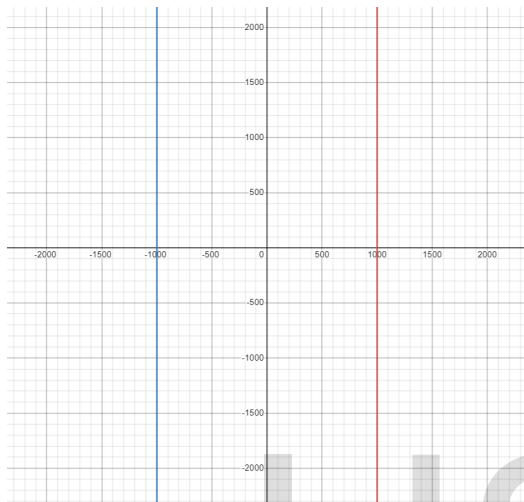


Figure 2: The graph of apparent path equation by DESMOS.

(ii) For $v = c = 300000000$,

$$\lambda = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{0} = \text{undefined}$$

And

$$M = \frac{yv}{c\sqrt{y^2 + \lambda^2 x^2}} = 0$$

So, y' becomes,

$$\Rightarrow y' = y \left(\frac{1}{\lambda} - M \right) = 0$$

And

$$x' = x \left(1 - \frac{yv}{c\sqrt{y^2 + \lambda^2 x^2}} \lambda \right) = x - y$$

Which means that all points on the wall appear to be lying on the line directly going out of our eye and intersecting the wall perpendicularly. But, the strange thing is that, the points on the right wall having $y < x$ will appear on the right side of that perpendicular and the points having $y > x$ will appear on the left side of the perpendicular.

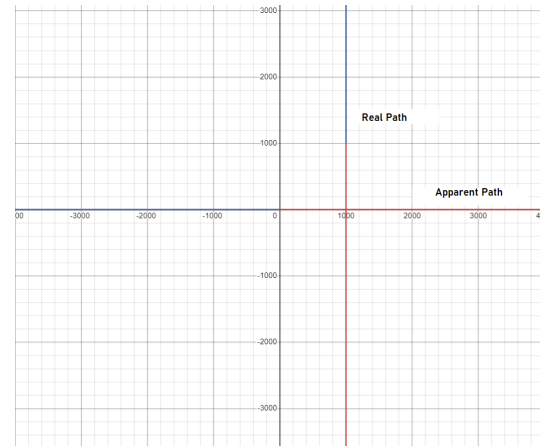


Figure 3: The graph of apparent path equation by DESMOS.

Similarly, the points on the left wall having $y < x$ will appear on the right side of the perpendicular and the points having $y > x$ will appear on the left side of perpendicular.

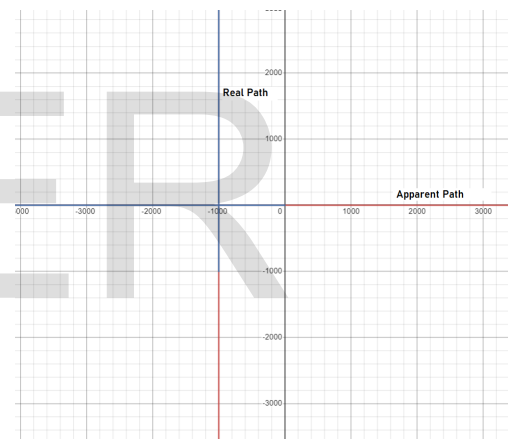


Figure 4: The graph of apparent path equation by DESMOS.

As the light rays from these points will come one after another, the light rays from behind a point will be blocked by the light ray from that point. So, we will be seeing the light ray from the point closest to our eye. Hence, we will not be able to see the world around us, like it never existed.

This is how the world would appear to a photon.

5 DISCUSSION

5.1 Interpretation of our findings

We have, from our formula, found out that:

1) when an observer is moving with respect to the space around it, the space ahead of it appears to be curving or warping in and the space behind him appears to be curving or warping away, as given in Fig.1.

- 2) When the observer is stationary the space appears as it appears to stationary observers.
- 3) When the observer is moving at a constant velocity of 300000000 m/s, then the space around us will be not visible to us.
- 4) For a given change in distance and relative velocity, the apparent path's equation is most influenced by a change in relative velocity.

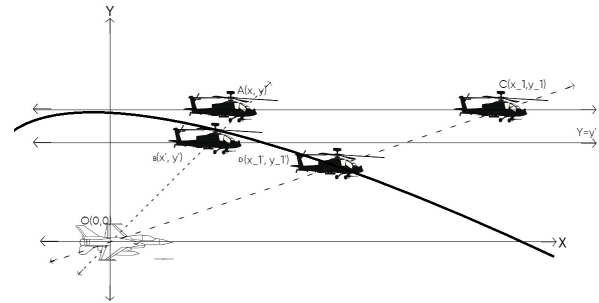


Figure 5:

Since Apache and Su-35 are moving at 50m/s and 800m/s respectively, therefore relative velocity of Apache is 750 m/s due negative x-axis w.r.t. Su-35. Hence, there will be contraction along the line of sight joining the Su-35 and Apache. So, Apache will appear to be closer than that it is. Let it be appearing at a position (x', y') and let the predicted one be appearing at (x'_1, y'_1) , as given in Fig.6.

Since, the fighter jet sees Apache at (x', y') , so it will calculate that apache will be moving along $Y = y'$ and hence the predicted position will be on the line $Y = y'$. Let that position be (x_2, y_2) , as given in Fig.7, where $y_2 = y'$. Now, the relative velocity of Apache w.r.t. Su-35 is 750 m/s, due negative x-axis. Let coordinates of be $A(2000, 2000)$. then, coordinates of $B(x', y')$ are,

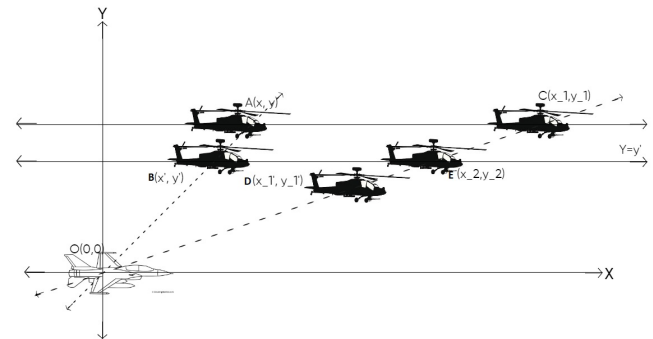


Figure 6:

$$x' = x \left(1 - \frac{yv}{c\sqrt{y^2 + \lambda^2 x^2}} \lambda \right)$$

$$y' = y \left(\frac{1}{\lambda} - \frac{yv}{c\sqrt{y^2 + \lambda^2 x^2}} \right)$$

After doing calculations, we found the values,

$$x' = 1999.996464 \quad \text{And} \quad y' = 1999.996464$$

and length of OA and OB are,

$$OA = \sqrt{x^2 + y^2} \quad \text{And} \quad OB = \sqrt{x'^2 + y'^2}$$

$$OA = 2828.427125 \quad \text{And} \quad OB = 2828.422124$$

So, the difference in original and appearing length is $OA - OB = .005m = 5mm$. and the error is of 0.00012 percent means 1 part in ten thousand.

Since, the line of sight's length gets on increasing, therefore the error also increases.

Since it is a very close combat fight the error is very low(still significant). If it is required to do high velocity targeting at long ranges, there can be a significant error.

Hence, this formula can be used to increase the accuracy of guns in fighter jets. It will prove to be very fruitful for "Future Sixth Generation Jets".

6 CONCLUSION

Earlier we had thought that "Lorentz Contraction" has no influence on the world, but as i have hypothesized that there is a anomalous effect of it on the world, is true. *The space appears to be contracted and expanded to an observer in motion, more accurately, the world ahead and behind the observer appears to be contracted and expanded along the line of sight, joining the points to the observer, to an observer in motion.*

We have also derived an equation relating the points on the apparent world viewed by the observer in motion to the world observed by the observer at rest.

A corollary of our equation(6), telling the perspective of photon says that, *"the world will not be visible to the photon like it never existed, which means that time for the photon stops down."* This exactly coincides with a claim, in Brian Greene's book "Elegant Universe", that time stops for a photon.

7 ACKNOWLEDGMENT

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